A being the value of a life annuity and r the rate of interest; and that the answer to the second is

$$\frac{a_{x+1}}{a_x}(1+r)+\frac{a_{x+2}}{a_x}(1+r)^2+\frac{a_{x+3}}{a_x}(1+r)^3+\ldots.$$

a_x denoting the number of persons living at any age x.

Mr. De Morgan also adds the following examples by the Northampton Table at 4 per cent., showing the average sums obtained in the two cases :—

Age.	1st Case.	2nd Case.	Age.	1st Case.	2nd Case.
20	49·4	98·3	45	27·2	38·0
25	44·7	82·2	50	23·2	30·7
30	40·2	68·6	55	19·7	24·5
35	35·7	56·8	60	17·0	19·2
40	31·3	46·7			

And Mr. Hardy enters into an elaborate investigation of *the proper method of determining the amount of an annuity forborne and improved at interest during the existence of a given life,* in a paper read before the Institute on the 26th January, 1857, and published in the April number of the *Journal* for that year, clearly demonstrating that the expressions above given are identical only when *the annuity is forborne and improved for a term of years certain,* or *when money bears no interest*—in fact, that one is the average of *present values,* the other of *amounts.*

Of Compound Interest. By DR. EDMUND HALLEY, *Royal Astronomer, Savilian Professor of Geometry, Oxford, and* F.R.S.*

A PRINCIPAL use of logarithms is to solve all the cases of compound interest which are not, without great difficulty, attainable by the rules of common arithmetic. But before we proceed to the practical part, it may, perhaps, not be improper to say something of the foundation or demonstration of the rules we are to give.

Therefore, let p be any sum of money forborne t times, r the rate of interest or produce of £1 and its interest in one time—that is, as 1 to r, so £1 to its amount after one year or other space of

* We republish this paper from *Sherwin's Mathematical Tables,* printed for W. and J. Mount, 1761; and, considering the celebrity of the writer, the ability displayed in the paper itself, and the comparative scarcity of the work from which it is taken, we believe we do no more than consult the wishes of our readers in causing it to reappear in the pages of this *Journal.*—ED. *A. M.*

time; and let m be the amount of the sum p forborne t times. Now, because in one year or time unity becomes r, by the same reason r will in another time become rr, and rr in a third time become r^3, &c., it appears that r^t, or r raised to the power whose index is the number of times, will be the amount of £1 forborne t times; and, therefore—

I. The amount $m=pr^t$; therefore multiply the logarithm of r by t, and to the product add the logarithm of p; the sum shall be the logarithm of m.

Example.—What is the amount of £15. 17*s.* 6*d.* forborne 12 years, at 6 per cent. per annum compound interest?

The number $1{\cdot}06=r$ . . . . . its log.	0·0253059
Which log. multiplied by $t=12$, the years, produces .	0·3036708
The principal sum £15. 17*s.* 6*d.*=£15·875=p, its log.	1·2007137
The amount £31. 18*s.* 10½*d.*=£31·94362=m, its log.	1·5043845

II. $r^t=\frac{m}{p}$; therefore, if from the logarithm of m the logarithm of p be subtracted, and the remainder be divided by t, the quotient is the logarithm of r.

Example.—What is the rate of compound interest when the sum of £15. 17*s.* 6*d.*, forborne 12 years, amounts to £31. 18*s.* 10½*d.*?

The amount £31. 18*s.* 10½*d.*=£31·94362=m, its log.	1·5043845
The principal sum £15. 17*s.* 6*d.*=£15·875=p, its log.	1·2007137
The remainder is log. of r^t	0·3036708
Which, divided by $t=12$, quotes log. of ($1{\cdot}06=r$) .	0·0253059

Therefore the rate is 6 per cent. per annum.

III. Because $r^t=\frac{m}{p}$; divide the difference of the logarithms of m and p by the logarithm of r; the quotient is t, or the time wherein the sum p will amount to m at the rate r.

Example.—In what time will the principal sum of £15. 17*s.* 6*d.* amount to £31. 18*s.* 10½*d.* at 6 per cent. per annum compound interest?

The amount £31. 18*s.* 10½*d.*=£31·94362=m, its log.	1·5043845
The principal sum £15. 17*s.* 6*d.*=£15·875=p, its log.	1·2007137
The remainder is log. of r^t	0·3036708

Which, divided by 0·0253059=log. of r, quotes 12 years for the time.

IV. $p=\frac{m}{r^t}$; therefore multiply the logarithm of r by t, and

subtract the product from the log. of m; the remainder shall be the log. of p, the principal sum.

Example.—What is the principal sum that in 12 years, at 6 per cent. per annum compound interest, will amount to £31. 18*s*. 10½*d*.?

The amount £31. 18*s*. 10½*d*.=£31·94362=m, its log.	1·5043845
The number 1·06=r . . . . . its log.	0·0253059
Which log., multiplied by t=12, the years, produces .	0·3036708
And subtracted from the log. of the amount—	
The remainder is log. of (£15·875=£15. 17*s*. 6*d*.=p)=	1·2007137

The four preceding rules are also readily deduced from the consideration of the rebate of money in this manner.

For if, in any time, r becomes 1, in the same time 1 becomes $\frac{1}{r}$, and in the second time $\frac{1}{r}$ becomes $\frac{1}{rr}$, and in the third $\frac{1}{rrr}$, &c.; so that, putting p the value or present worth of any sum m payable after t times, at the rate of r to 1:

I. The sum $m=pr^t$; therefore multiply the log. of r by t, and to the product add the log. of p, the sum shall be log. of m sought.

II. $r^t=\frac{m}{p}$; therefore from the log. of m subtract the log. of p, and divide the remainder by t; the quotient will be the log. of r.

III. Since $r^t=\frac{m}{p}$, divide the difference of the logs. of m and p by the log. of r; the quotient shall be t, the number of years.

IV. $p=\frac{m}{r^t}$; therefore multiply the log. of r by t, and subtract the product from the log. of m; the remainder will be the log. of p, which finds the value of any sum of money payable after any time assigned.

The logarithms are also serviceable to resolve all questions concerning the amount or present worth of annuities not paid as due, or purchased to be paid for in time to come.

Let a be an annuity or yearly pension whose successive amounts for times past are ar^t, and whose present values are $\frac{a}{r^t}$ successively, by what goes before; and the series, &c., ar^5, ar^4, ar^3, ar^2, ar, a, $\frac{a}{r}$, $\frac{a}{r^2}$, $\frac{a}{r^3}$, $\frac{a}{r^4}$, $\frac{a}{r^5}$, &c., will be a rank of mean proportionals continued infinitely in the ratio of r to 1. Now, the sum of all the consequents, or of the whole infinite series, will be to the said

sum increased by the next greater term (or the sum of all the antecedents) as 1 to r, by *Euclid*, v. 12; wherefore, putting y for the said sum of the consequents, ry will be equal to $y+ar^t$, the sum of the antecedents; and $ry-y=ar^t$; and, therefore, $\frac{ar^t}{r-1}$ will be equal to y, the sum of all our mean proportionals whereof ar^{t-1} is the greatest; and, by the same rule, $\frac{a}{r-1}$ will be the sum of all the terms whereof $\frac{a}{r}$ is the greatest. So that if we subtract $\frac{a}{r-1}$ from $\frac{ar^t}{r-1}$, the difference will be the sum of all the terms whereof ar^{t-1} is the greatest and a the least, their number being t, which sum we will call z; therefore z (the amount of the annuity of a forborne t times at the rate r) $=\frac{ar^t-a}{r-1}$.

I. *The annuity* (a), *rate of interest* (r), *and time* (t), *being given, to find* (z) *the amount.*

From the log. of a subtract the log. of $\overline{r-1}$, and to the remainder add the log. of r^t; from the number answering to this last sum, subtract the number answering to the remainder; the difference shall be z, the amount sought.

Example.—What will an annuity of £34·4, forborne 12½ years, amount to at 6 per cent. per annum?

$a=34{\cdot}4$	$\text{L. }a=1{\cdot}5365584$	$\text{L. }r=0{\cdot}0253059$
$r-1=0{\cdot}06$	$\text{L. }\overline{r-1}=8{\cdot}7781513$	$t=\ .\ .\ 12{\cdot}5$
$\frac{a}{r-1}=573{\cdot}3333$	$\text{L. }\frac{a}{r-1}=2{\cdot}7584071$	1265295
		506118
	$\text{L. }r^t=0{\cdot}3163237$	253059
$\frac{ar^t}{r-1}=1187{\cdot}7660$	$\text{L. }\frac{ar^t}{r-1}=3{\cdot}0747308$	$\text{L. }r^t=0{\cdot}3163237{\cdot}5$

$$z=£614{\cdot}4327=\frac{ar^t}{r-1}-\frac{a}{r-1}=\text{the amount.}$$

II. *The annuity* (a), *its amount* (z), *and the rate of interest* (r), *being given, to find* (t) *the time.*

By the foregoing, $\frac{ar^t-a}{r-1}=z$, therefore $z+\frac{a}{r-1}=\frac{a}{r-1}\times r^t$; wherefore, from the log. of a subtract the log. $\overline{r-1}$; to the number answering to the remainder add the given amount, and from the log. of the sum subtract the afore-found remainder; this

second remainder, divided by the log. of r, will quote the time required.

Example.—In what time will an annuity of £34·4 amount to £614·4327 at the rate of 6 per cent. per annum?

$a=34{\cdot}4$	L. $a=1{\cdot}5365584$
$r-1=0{\cdot}06$	L. $\overline{r-1}=8{\cdot}7781513$
$\dfrac{a}{r-1}=573{\cdot}3333$	L. $\dfrac{a}{r-1}=2{\cdot}7584071$
$z=614{\cdot}4327$	
$\dfrac{ar^t}{r-1}=1187{\cdot}7660$	L. $\dfrac{ar^t}{r-1}=3{\cdot}0747308$

L. $r=0{\cdot}0253059)0{\cdot}3163237(12{\cdot}5$ years $=t$.

III. *The amount* (z), *rate of interest* (r), *and time* (t), *being given, to find* (a) *the annuity.*

The former equation being reduced, $a=\dfrac{z\times\overline{r-1}}{r^t-1}$; wherefore, to the log. of the amount z, add the log. of $\overline{r-1}$, and from the sum subtract the log. of $\overline{r^t-1}$; the remainder is the log. of a.

Example.—An annuity forborne $12\frac{1}{2}$ years amounts, at 6 per cent. per annum, to the sum of £614·4327, how much is that annuity?

$z=614{\cdot}4327$	L. $z=2{\cdot}7884743$	L. $r=0{\cdot}0253059$
$r-1=0{\cdot}06$	L. $\overline{r-1}=8{\cdot}7781513$	$t=\ .\ .\ 12{\cdot}5$
$r^t=2{\cdot}071685$	$1{\cdot}5666256$	L. $r^t=0{\cdot}3163237$
$r^t-1=1{\cdot}071685$	L. $\overline{r^t-1}=0{\cdot}0300671$	

L. £34·4 the annuity $=$ L. $a=1{\cdot}5365585$

IV. *The annuity* (a), *time* (t), *and amount* (z), *being given, to find* (r) *the rate.*

In order to find r, the former equation is reduced to $\dfrac{z}{a}-1=\dfrac{z}{a}r-r^t$, or, in our present case, $16{\cdot}8614=17{\cdot}8614r-r^t$, which is so affected as not readily to be resolved by the general method for resolution of equations, unless we can first approach it by some other means; for which purpose take the following rule, which will suffice where great exactness is not required.

Let $\overline{\dfrac{z}{at}}\Big|^{\frac{t-1}{2}}=1+y$, and let $\dfrac{6}{t+1}=b$. I say that $\overline{bb+2by}\Big|^{\frac{1}{2}}-b$ is exceeding near the increase of the rate, or $r-1$.

Wherefore take the log. of the amount, and the complements of the logarithms of the time and annuity; the sum (abating 2 in

the tens place of the index) divide by $\frac{1}{2}\times\overline{t-1}$; the quotient shall be the log. of $1+y$. Then divide 6 by $t+1$, and to b (the quotient) add twice y, and to the log. of b add the log. of $\overline{b+2y}$; half the sum shall be the log. of $\overline{bb+2by}|^{\frac{1}{2}}$, from which square root subtract b; the residue will be very near the increase, or $r-1$; and, adding 1, r is found. If great exactness be desired, let r thus found be assumed, and $\frac{z}{a}r-r^t$, compared with $\frac{z}{a}-1$, will always be greater than it; and dividing the excess by $tr^{t-1}-\frac{z}{a}$, the quotient added to r shall verify as many more figures in the rate as were true in the assumed r.

Example.—An annuity of £34·4, forborne $12\frac{1}{2}$ years, amounts to £614·4328, required the rate of interest allowed.

$z=614{\cdot}4327$	L. $z=2{\cdot}7884743$
$t=12{\cdot}5$	co. L. $t=8{\cdot}9030900$
$a=34{\cdot}4$	co. L. $a=8{\cdot}4634416$

$\frac{1}{2}\times\overline{t-1}=5{\cdot}75)0{\cdot}1550059(0{\cdot}0269575=$ L. $\overline{1+y}$, and $1+y=1{\cdot}064039$

$b=\frac{6}{t+1}=\frac{6}{13{\cdot}5}$	$=0{\cdot}4444444$	$2y=0{\cdot}128078$
$2y$	$0{\cdot}1280710$	L. $b=9{\cdot}6478175$
$b+2y$	$0{\cdot}5725224$	L. $\overline{b+2y}=9{\cdot}7577936$
		$19{\cdot}4056111$
$\overline{bb+2by}|^{\frac{1}{2}}$ . . .	$0{\cdot}5044353$	L. $\overline{bb+2by}|\frac{1}{2}=9{\cdot}7028055$
b	$0{\cdot}4444444$	
$r-1$	$0{\cdot}0599909$	

Therefore the rate of interest sought is 6 per cent. per annum.

After the same manner the four cases relating to the purchase of annuities are readily solved by logarithms, and the theorems discovered with the same ease; for a, $\frac{a}{r}$, $\frac{a}{rr}$, $\frac{a}{r^3}$, $\frac{a}{r^4}$, &c., being a scale of mean proportionals in the ratio of r to 1, put y for the sum of all the consequents infinitely continued, whereof $\frac{a}{r}$ is the first, and that sum will be to the sum of all the antecedents whereof a is the first, as 1 to r (that is, $1:r::y:ry$), so that $ry=y+a$, and $\frac{a}{r-1}$ will be equal to y, the value of the fee, or the sum of all the mean proportionals less than a. And, by the same rule, $\frac{a}{r^t\times\overline{r-1}}$ will be the sum of all the means less than $\frac{a}{r^t}$, or the value of the reversion; and, subtracting the one sum from the other,

$\frac{a}{r-1} - \frac{a}{r^t \times \overline{r-1}}$ will be equal to z, the sum of all the means, whereof $\frac{a}{r}$ is the greatest and $\frac{a}{r^t}$ the least.

I. *The annuity* (a), *time* (t), *and rate of interest* (r), *being given, to find* (z) *the present value.*

The present value $z = \frac{a}{r-1} - \frac{a}{r^t \times \overline{r-1}}$; therefore, from log. of the annuity subtract the log. of $\overline{r-1}$, and from the residue subtract the log. of r^t; the difference of the numbers answering to the two remainders is the present value sought.

Example.—What is £70 per annum, to continue 59 years, worth in present money, at the rate of 5 per cent. per annum?

$a=70$	L. a=1·8450980	L. r=0·0211893
$r-1$=0·05	L. $\overline{r-1}$=8 6989700	t= . . . 59
Fee=£1400	L. fee=3·1461280	1907037
Reversion, £78·6972	L. r^t=1·2501687	1059465
	L. rever.=1·8959593	L. r^t=1·2501687

z=£1321·3028, the present value sought.

II. *The annuity* (a), *present value* (z), *and rate of interest* (r), *being given, to find* (t) *the time.*

Now, r^t will be equal to $\frac{a}{r-1}$ (or the fee) divided by the value of the reversion—that is, by $\frac{a}{r-1} - z$; wherefore, from the log. of the annuity subtract the log. of $\overline{r-1}$; the number answering to the remainder will be the value of the fee. From the fee subtract the present worth, the residue is the value of the reversion. Take the log. of the reversion from the log. of the fee, and divide the residue by the log. of r; the quotient will be t, the number of years sought.

Example.—In what time will an annuity of £70 per annum pay off a debt of £1321·3028, allowing the creditor 5 per cent. per annum?

$a=70$	L. a=1·8450980
$r-1$=0·05	L. $\overline{r-1}$=8·6989700
Fee=1400	L. fee=3·1461280
z—1321·3028	
Reversion, 78·6972	L. rev.=1·8959593

L. r=0·0211893)1·2501687(59 years=t.

III. *The present value* (z), *rate of interest* (r), *and time* (t), *being given, to find* (a) *the annuity.*

The former equation may be reduced to this proportion, as $1-\frac{1}{r^t}$ to z, so is $r-1$ to a, the annuity sought.

Wherefore, to the complement of the log. of $\overline{1-\frac{1}{r^t}}$, add the logs. of z and of $\overline{r-1}$, the sum shall be the log. of a.

Example.—What annuity, to continue 59 years, can be purchased for £1321·3028, at the rate of 5 per cent: per annum?

$\frac{1}{r^t}=0·0562123$		L. $r=0·0211893$
		$t= \ldots 59$
$1-\frac{1}{r^t}=0·9437877$	co. L. $\overline{1-\frac{1}{r^t}}=0·0251257$	L. $r^t=1·2501687$
$z=1321·3028$	L. z . . . 3·1210023	L. $\frac{1}{r}=8·7498313$
$r-1=0·05$	L. $\overline{r-1}=8·6989700$	
	L. (£70$=a$ the annuity sought)$=1·8450980$	

IV. *The annuity* (a), *present value* (z), *and time* (t), *being given, to find* (r) *the rate of interest.*

This problem being more difficult than appears at first sight, and requiring the solution of this equation, $\frac{a}{z}=\frac{z+a}{z}r^t-\frac{t+1}{r}$, to which it is reduced, there must be applied some method of approaching the root r, which is by no means evident; and that approximation, as the number of years and rate are greater or less, cannot properly be obtained by one general rule, but rather by two, according as the value of the reversion is greater or less.

If the number of years be great (as suppose 40 or upwards), and especially if the rate of interest be high, $1+\frac{a}{z}$ will be nearly the rate; or, more accurately, $\frac{z+a}{z}-\frac{z^t}{z+a}\times\frac{a}{z}$. Call it r, and $\frac{a}{r^t\times\overline{r-1}}$ will be exceeding near the value of the reversion, which let be x; then $1+\frac{a}{z+x}$ shall approach the true rate sufficiently. But if greater exactness be desired, by repeating this process it will be obtained. Hence this rule:—from the log. of a, and also from the log. of $z+a$, take the log. of z; this latter remainder

shall be nearly the log. of the rate. Multiply that log. by t, and the complement of the product add to the first remainder; the decimal fraction answering to the sum taken from the former rate shall give a more correct rate. With this rate seek x, the reversion after the time given, which add to z; then to the complement of the log. of $\overline{z+x}$ add the log. of a; the sum shall be the log. of the increase, or of $\overline{r-1}$, sufficiently near.

Example.—If £1321·3028 is paid for an annuity of £70 per annum for 59 years to come, what is the rate of interest allowed the purchaser?

$a=70$	L. $a=1\cdot8450980$	L. $\overline{z+a}=3\cdot1434217$
$z=1321\cdot3028$	L. $z=3\cdot1210023$	L. z . . $3\cdot1210023$
$\frac{z+a}{z}=1\cdot052978$	$8\cdot7240957$	L. $\frac{z+a}{z}=0\cdot0224194$
	co. tL. $\frac{z+a}{z}=8\cdot6772554$	
		t 59
Log. of 0·00252	$7\cdot3013511$	
		2017746
First $r=1\cdot050458$	L. $r=0\cdot0213787$	1120970
	$t=$ 59	
$z=1321\cdot3028$	1924083	tL. $\frac{z+a}{z}=1\cdot3227446$
$x=\ 76\cdot002$	1068935	
		co. L. $r'=8\cdot7386567$
$z+x=1397\cdot3048$ the fee	L. $r'=1\cdot2613433$	co. L. $\overline{r-1}=1\cdot2970700$
	co. L. $\overline{z+x}=6\cdot8547088$	L. $a=1\cdot8450980$
	L. a . . $1\cdot8450980$	
		L. $x=1\cdot8808247$
$r-1=0\cdot05096$	L. $\overline{r-1}=8\cdot6998068$	

So that the rate of interest sought is 5 per cent. per annum.

If the number of years be small, the aforesaid rule will avail little. In this case it will be requisite to approach the rate thus:— Let $\frac{t+1}{2}$ be the index of a root of $\frac{at}{z}$; from which root subtract 1, and the remainder call y, and let $\frac{6}{t-1}$ be called b. I say that $b-\overline{bb-2by}|^{\frac{1}{2}}$ is sufficiently near to $r-1$, and will be still nearer the truth as the number of years is smaller; and that the error will be always in excess. Hence the rule:—divide the log. of $\frac{at}{z}$ by $\frac{t+1}{2}$, and from the number answering to the quotient subtract 1; double the remainder, and subtract it from b—that is,

from the quotient of 6 divided by $t-1$; to the logarithm of this remainder add the log. of b; then the number answering to half the sum of those logarithms taken from b will leave $r-1$, the increase of the rate sought.

Example.—An annuity of £20 per annum, to continue 21 years, is sold for £220: required the rate of interest allowed the purchaser.

$$\begin{array}{ll} a=20 & \text{L.}\,a=1{\cdot}3010300 \\ t=21 & \text{L.}\,t=1{\cdot}3222193 \\ z=220 & \text{co. L.}\,z=7{\cdot}6575773 \end{array}$$

$$\tfrac{1}{2}\times\overline{t+1}=11)0{\cdot}2808266(0{\cdot}0255297=\text{L.}\,\overline{1+y},\text{ and }1+y=1{\cdot}060546$$

$$2y=0{\cdot}121092$$

$$b=\frac{6}{t-1}=\frac{6}{20}=0{\cdot}3 \qquad \begin{array}{r} \text{L.}\,b=9{\cdot}4771213 \\ \text{L.}\,\overline{b-2y}=9{\cdot}2526298 \end{array}$$

$$18{\cdot}7297511$$

$$\text{L.}\,\overline{bb-2by}|^{\frac{1}{2}}=9{\cdot}3648755$$

$$\begin{array}{ll} 2y \;\ldots\ldots & 0{\cdot}121092 \\ b-2y \;\ldots\ldots & 0{\cdot}178908 \\ \overline{bb-2by}|^{\frac{1}{2}} \;\ldots\ldots & 0{\cdot}231673 \end{array}$$

Now, $b-\overline{bb-2by}|^{\frac{1}{2}}=0{\cdot}068327=r-1$

Therefore $r=1{\cdot}068327$, the rate sought.

The rate r thus found is always some small matter too big, the true rate being 1·06814; but as the number of years are fewer, the error becomes insensible. If greater exactness be required, it will be easy by the general method for the resolution of equations having so near an approximation, to prosecute this inquiry as far as you please; but this seems abundantly sufficient for use, which is our principal design in this place.

Lastly, by way of corollary to the former: let it be required to find the rate of interest allowed the purchaser when he pays a sum z, for an annuity a, wherein he has already a term t, to have it prolonged for a certain time x.

Example.—An annuity of £20 per annum being in possession for the term of 21 years, and for £40 paid down it can be prolonged for 10 years more, or to 31 years; what is the rate of interest required?

Put $T=2t+x+1$, and $\frac{1}{2}T$ shall be the index of a root of $\frac{ax}{z}$. Let $\overline{\frac{ax}{z}}|^{\frac{1}{2}T}$ be equal to $1+y$, and $\frac{6T+6}{xx}=b$. I say $r-1$ is very near to $b-\overline{bb-2by}|^{\frac{1}{2}}$.

$a=20$ $\quad$ L. $a=1{\cdot}3010300$
$x=10$ $\quad$ L. $x=1{\cdot}0000000$
$z=40$ $\quad$ co. L. $z=8{\cdot}3979400$

$\frac{1}{2}\mathrm{T}=26{\cdot}5)0{\cdot}6989700(0{\cdot}0263762=\mathrm{L.}\,\overline{1+y}$, and $1+y=1{\cdot}062616$

$2y=0{\cdot}125232$

$$b=\frac{6\mathrm{T}+6}{xx}=3{\cdot}24$$

L. $b=0{\cdot}5105450$
L. $\overline{b-2y}=0{\cdot}4934257$

$2y$ . . . . $0{\cdot}125232$

$1{\cdot}0039707$

$\overline{bb-2by}|^{\frac{1}{2}}=3{\cdot}176767$ $\quad$ L. $\overline{bb-2by}|^{\frac{1}{2}}=0{\cdot}5019853$

$b-\overline{bb-2by}|^{\frac{1}{2}}=0{\cdot}063233=r-1$

Therefore $r=1{\cdot}063233$, the rate sought,

as will be readily proved by seeking the value of the reversion of an annuity of £20 per annum for 10 years after 21, at the rate of 1·063233 per cent. per annum. (*See* the Work.)

$a=20$	L. $a=1{\cdot}3010300$	L. $r=0{\cdot}0266284$
$r-1=0{\cdot}063233$	co. L. $\overline{r-1}=1{\cdot}1990562$	21
	co. L. $r^t=9{\cdot}4408036$	266284
Rever. £87·275	L. rev. $=1{\cdot}9408898$	532568
	co. L. $r^x=9{\cdot}7337160$	L. $r^t=0{\cdot}5591964$
Rever. £47·2722	L. rev. $=1{\cdot}6746058$	

£40·0028, the value sought.

Thus it appears that £40 and about three farthings is the true value of the difference of the reversions at the rate of interest before found; by which it may be judged how near an approximation the foregoing rule affords towards finding the rate of interest, when the value of an annuity for a term of years to commence after such a distant time is proposed.